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Pricing Equity Default Swaps under the Jump to Default Extended CEV Model

Rafael Mendoza-Arriaga · Vadim Linetsky

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Abstract Equity default swaps (EDS) are hybrid credit-equity products that provide a bridge from credit default swaps (CDS) to equity derivatives with barriers. This paper develops an analytical solution to the EDS pricing problem under the Jump-to-Default Extended Constant Elasticity Variance Model (JDCEV) of Carr and Linetsky. Mathematically, we obtain an analytical solution to the first passage time problem for the JDCEV diffusion process with killing. In particular, we obtain analytical results for the present values of the protection payoff at the triggering event, periodic premium payments up to the triggering event, and the interest accrued from the previous periodic premium payment up to the triggering event, and determine arbitrage-free equity default swap rates and compare them with CDS rates. Generally, the EDS rate is strictly greater than the corresponding CDS rate. However, when the triggering barrier is set to be a low percentage of the initial stock price and the volatility of the underlying firm's stock price is moderate, the EDS rates and CDS rates are quite close. Given the current movement to list CDS contracts on organized derivatives exchanges to alleviate the problems with the counterparty risk and the opacity of over-the-counter CDS trading, we argue that EDS contracts with low triggering barriers may prove to be an interesting alternative to CDS contracts offering some advantages due to the unambiguity and transparency of the triggering event based on the observable stock price.

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R. Mendoza-Arriaga
Information, Risk, & Operations Management Dept. (IROM). McCombs School of Business. The University of Texas at Austin. CBA 5.202, B6500. 1 University Station, Austin, TX 78712, Phone: (512) 471 5824. E-mail: rafael.mendoza-arriaga@mccombs.utexas.edu

V. Linetsky
Department of Industrial Engineering and Management Sciences, McCormick School of Engineering and Applied Sciences, Northwestern University, 2145 Sheridan Road, Evanston, IL 60208, Phone: (847) 491 2084, Web: http://users.iems.northwestern.edu/∼linetsky. E-mail: linetsky@iems.northwestern.edu

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1 Introduction

Equity default swaps (EDS) are a class of hybrid credit-equity products that provide a bridge from credit default swaps (CDS) to equity derivatives with barriers. JP Morgan's Odysseus deal (2003) was the first major transaction in which EDS were included as synthetic collateral for collateralized debt obligations (CDO). The Odysseus CDO consisted of 90% CDS and 10% EDS on a notional amount of 1.2 billion Euro (see Weidner et al. [25]). Moody's assigned it a credit rating in December 2003. In 2004 Daiwa Securities originated the first rated CDO of pure EDS with the notional of 45 billion yen. While the on-going crisis in the credit markets worldwide puts into question the viability of some of the over-the-counter credit derivatives, given the significant counterparty risks involved, an alternative point of view has emerged that credit derivatives, such as CDS, should be traded on organized derivatives exchanges to gain transparency and the benefits of the clearing house guaranteeing transactions to remove counterparty risk. Indeed, in some recent cases CDS protection writers ended up with weaker credit ratings than some of the names they have written protection on, in part due to the heavy losses they have sustained on settling CDS contracts they have written on other names. As we write this, the negotiations are on-going with regard to creating the CDS exchange (see Gutierrez [15]). Notably, EDS provide a bridge from CDS to equity derivatives products, with the transparency of the latter based on the observable stock price. In this sense, standardized and transparent EDS contracts might be a more natural product for derivatives exchanges than CDS contracts.

Since the Odysseus deal, there has been an on-going debate in the marketplace whether EDS is truly a credit-like product, or just another exotic equity derivative. For instance, Société Générale's *Yield Enhancement Strategies* (YES) are EDS-like products that have been marketed for a long time (see Sawyer [22]). Some see EDS contracts as a variation of deep-out-of-the-money American digital options for which the premium is paid in installments over time up to the trigger event, thus viewing EDS as a type of exotic equity option. Other authors state that EDS close the gap between equity and credit since they are structurally very similar to CDS. An EDS contract delivers a protection payment if the reference entity's stock price drops below a pre-specified lower barrier level, typically set at 30% of the stock price at the contract inception. The *triggering event* is thus a severe stock price drop of 70% or more from the initial level. In exchange, the EDS buyer makes periodic premium payments that accrue at the contracted fixed swap rate up to the triggering event or the final contract maturity, whichever comes first. The CDS thus arises in the limit of the barrier tending towards zero, since in the event of the firm's default on its debt the stock price would be traded near zero. If the absolute priority holds, then the stock would be worth exactly zero. Violations of absolutely priority rules may result in the firm's stock trading for some small positive value after the bond default occurs. In any case, the stock price of a firm in default on its bonds is typically very low.

The EDS premium rate is strictly greater than the CDS rate since the CDS triggering event is also an EDS triggering event, but it is possible that the stock price may suffer a drop of 70% of the initial price that will trigger the EDS' protection payment without having a debt default event triggering CDS payoffs. This fact attracts some investors seeking credit protection. These investors argue that empirical evidence supports a positive relationship between severe stock price declines and the increased probability of default of the reference firm. Empirical studies by Jobst and de Servigny [16, 17] provide some insightful results about the credit-equity nature of EDS. Jobst and de Servigny [16] empirically estimate the *default probability* and the *equity event probability*. The authors look at a subset of U.S. firms in the Compustat database for which S&P credit rating and other risk factors are available. For each time horizon (1 and 5 years), they observe the number of firms whose stock price has experienced 70%-90% drops from the initial stock price (relative to the beginning of the observation period) by retrieving the monthly minimum stock price for each period. They use this result to estimate the *equity event probability*. Similarly, in order to estimate the *default probability*, they look at the number of firms that defaulted during the same period of time. Subsequently, they employ these estimated probabilities as dependent variates in a linear logit model to analyze the explanatory power of various credit and equity factors at different triggering levels returning consistent results across different maturities. They notice that credit ratings and historical volatilities have the highest explanatory power among the other factors considered. This is true in particular for EDS with triggering levels at 50% and below (i.e., when the stock price needs to drop 50% or more to trigger the EDS payoff). However, for barrier levels above 50%, the equity related factors become more significant. Thus, it can be inferred that for low triggering levels the EDS can be seen as a credit-like instrument, but for higher triggering levels the EDS behaves more as an equity-like instrument. This remark is also supported by their Kendall's tau analysis in which they compare the frequency with which the EDS triggering event also implied the CDS triggering event (i.e., default). They show empirically that for low triggering levels (10%-30%) it is quite common that the EDS and CDS' protection payments will be triggered within the same period of time with frequency greater than 90%, but this is significantly less frequent for triggering levels above 50%. In Jobst and de Servigny [17], the event correlation was analyzed. They found that EDS triggering events have a significantly higher correlation than default events and, as expected, this correlation increases when increasing the barrier level, which can be explained by the fact that adverse market conditions common to firms in a particular sector(s) would be sufficient to trigger the EDS payoff if the barrier level is high enough. In addition, the EDS correlation is lower for investment grade firms than for non-investment grade firms, which supports their previous results that show that credit ratings and volatility have a high explanatory power.

Notwithstanding the previously discussed similarities between the EDS and CDS, there are some advantages inherent in the EDS contract design. Since CDS are triggered only in the event of default, CDS are indifferent to credit downgrades. In practice, prior to default, the firm's debt will experience a series of credit downgrades, while the stock price establishes new lows over time. EDS triggering events can be viewed as proxies for credit downgrades. Another important advantage of the EDS structure is the transparency with which the EDS payoff is triggered based on the fully observable stock price. While establishing a CDS credit event may not be an entirely simple matter, the EDS triggering event is unambiguous and fully observable. This transparency and unambiguity can be particularly important for exchange-traded contracts.

Several recent papers consider EDS pricing. Picone [20] assumes that the stock price follows a GARCH model. Asmussen, Madan and Pistorius [4] obtain an analytical approximation for EDS rates when the stock price follows a CGMY Lévy process by approximating the Lévy measure by hyperexponentials. They obtain analytical solutions for first passage times for the approximating process, calibrate the parameters of the CGMY model to GM and Ford options, and compute the EDS rates with triggering barriers at 30% of the initial stock price in the calibrated model. They show that the market observed CDS rates for both firms are higher than the modeled EDS rates. A possible explanation for this is that the CGMY process never hits zero, and thus excludes the possibility of default since the equity never becomes worthless in this model. As a result, if one were to take the limit of the EDS rates in the CGMY model as the triggering barrier tends to zero, one would *not* recover any positive CDS rate, but instead would obtain zero CDS rates in the limit. In our view, conventional models used in equity derivatives modeling such as stochastic volatility models and Lévy processes where the stock price never hits zero do not provide an adequate modeling framework to investigate the relationship between CDS and EDS contracts.[1]

In contrast, Albanese & Chen [3], Atlan and Leblanc [6,5], and Campi and Sbuelz [10] value EDS in the CEV model, where the stock price can hit zero via diffusion. Atlan and Leblanc also consider the Constant Elasticity of Stochastic Variance (CESV) process and other absolutely continuous time changes of the CEV process for valuing CDS in models with stochastic volatility. In these models the EDS rate is monotone in the triggering level and, in particular, the EDS rate is strictly greater than the CDS rate that is obtained in the limit of the zero triggering level. However, in the CEV model the event of default can only happen via continuous diffusion of the stock price towards zero. There is no element of surprise – no possibility of a jump to default from a positive stock value. It is well known that such models produce unrealistically low credit spreads for shorter maturity debt instruments and CDS. To remedy this shortcoming, Campi, Polbennikov and Sbuelz [9] value CDS in a model where the CEV diffusion process is killed at the first jump time of an independent Poisson process with constant intensity. However, the default intensity in their model is independent of the stock price, implying that the default is as likely when the stock price is very high, as it is when the stock price is very low. This assumption con-

[1] In our discussion of EDS and CDS contracts here and throughout the paper, we assume that the recovery and the corresponding loss-given-default payoff is a fixed percentage of the notional amount of the swap contract and is the same for EDS and CDS contracts. Thus, we are referring to CDS contracts with fixed loss-given-default payoffs specified in the contract. CDS contracts with loss-given-default amounts determined from the market prices of referenced bonds at the time of default are more complicated, since one needs to model stochastic recovery/LGD.

tradicts the accumulated empirical evidence that suggests a close link between the default probability and the stock price, as well as the volatility level.

To overcome these difficulties and to develop a unified credit-equity modeling framework, Carr and Linetsky [11] propose modeling the stock price as a jump-to-default extended CEV process where prior to default the stock price follows a diffusion process with constant elasticity of variance. The jump to default event has the intensity that is an affine function of the local variance. This links the default intensity, the stock volatility, and the stock price itself, since in the CEV model the local volatility is the (negative) power of the stock price. Thus, the JDCEV model exhibits both the volatility skews observed in the equity options markets and the links between default probability and the stock price and stock volatility well known in the credit markets. Carr and Linetsky obtain closed-form solutions for European options, survival probability, and corporate bonds in the JDCEV model.

The main contribution of this paper is the solution of the EDS pricing problem in the JDCEV model. We argue that the hybrid credit-equity JDCEV model is well suited to analyze hybrid credit-equity products such as EDS and overcomes the problems experienced with other models in the literature discussed above. Indeed, the event of default (CDS triggering event) in the JDCEV model is either the first hitting of zero for the CEV diffusion or a jump-to-default from a strictly positive value with the intensity given by the affine function of the CEV local variance, whichever comes first. The EDS triggering event is either the first hitting time of a triggering barrier for the CEV diffusion or a jump-to-default that takes the stock from a positive value above the barrier down to zero, whichever comes first. As such, the EDS rate is monotone in the triggering level, and the CDS rate naturally obtains from the EDS rate in the limit of the triggering level tending to zero. We stress that mathematically the first passage time problem for the JDCEV process is significantly more difficult than the European option pricing problem solved in Carr and Linetsky and requires an entirely different technical approach. We solve this problem by first computing Laplace transforms of various expectations involving the first passage time by explicitly constructing the resolvent operator for the JDCEV process in the domain above the barrier and then inverting the Laplace transforms analytically. We are able to obtain closed form solutions for the present values of the EDS protection payment, periodic premium payments, and the payment of accrued interest from the last periodic payment up to the triggering event.

The remainder of this paper is organized as follows. Section 2 describes the EDS model set up and describes how to value the EDS protection payment, the periodic premium payments, and the accrued interest payment up to the triggering event in the jump-to-default extended diffusion framework. In particular, it reduces all the valuation problems to a collection of expectations involving various functionals defined on paths of the underlying diffusion process. Section 3 develops analytical solutions for all the expectations necessary to value EDS in the JDCEV model. Section 4 shows the expressions of the EDS protection payment, the periodic premium payments, and the accrued interest payment up to the triggering event in the JDCEV model. Section 5 provides numerical examples and analysis of EDS rates. It investigates the impact of the triggering barrier level on the EDS rates, and compares them to the CDS rates that arise in the limit of the triggering barrier tending to zero. The proofs are included in the Appendix.

2 EDS Model Setup

We follow the model setup of Carr and Linetsky [11] and take as given a probability space $(\Omega, \mathcal{G}, \mathbb{Q})$ carrying a standard Brownian motion B and an exponential random variable $e \sim Exp\,(1)$ independent of B. We assume frictionless markets, no arbitrage, and the existence of an Equivalent Martingale Measure (EMM) $\mathbb{Q}$. Prior to default, the stock price is assumed to follow a diffusion process X under the EMM that solves the following stochastic differential equation:

$$dX_t = [r - q + h(X_t)]\, X_t dt + \sigma\,(X_t)\, X_t dB_t, \quad X_0 = x > 0 \tag{2.1}$$

where $r, q \geq 0$, $\sigma\,(X_t) \geq 0$, and $h\,(X_t) \geq 0$ are the risk-free rate, dividend yield, instantaneous stock volatility, and the state dependent jump-to-default intensity (hazard rate).

The event of default is formally defined as the equity becoming worthless, i.e., the stock price dropping to zero. It can happen in one of two ways. Either the diffusion process X hits zero via diffusion, or a jump-to-default occurs that takes the stock price from some positive value to zero, whichever comes first. Formally, the default time (or killing time) is $\zeta = T_0 \wedge \tilde{\zeta}$, where $T_0 = \inf\{t \geq 0 : X_t = 0\}$ is the first hitting time of zero for the diffusion process (2.1) and $\tilde{\zeta} = \inf\{t \geq 0 : H_t \geq e\}$ is the jump-to-default time with intensity (hazard rate) $h(X_t)$ and the integrated hazard process $H_t = \int_0^t h(X_u)du$.

The EDS contract delivers a protection payment to the EDS buyer at the time of the triggering event defined as the stock price decline below a pre-specified lower triggering barrier level. In exchange the EDS buyer makes periodic premium payments at time intervals Δ at the equity default swap rate up to the triggering event or the final maturity, whichever comes first. If the triggering event occurs mid-period between the two premium payments, the buyer pays the accrued interest from the time of the last premium payment up to the time of the triggering event. The protection payment is the specified percentage $(1 - \mathfrak{r})$ of the EDS notional amount $\mathcal{N}$ (by analogy with the CDS, here $\mathfrak{r}$ is the “recovery rate" and $1 - \mathfrak{r}$ is the “loss-given-default", or rather the “loss-given-the-triggering barrier crossing" event, that the EDS pays out to the EDS buyer). The valuation problem is to determine the arbitrage-free swap rate ϱ so that the present value of the EDS contract is zero at the contract inception time (time zero). This swap rate equates the present value of the protection payoff to the present value of all the premium payments (including possible accrued interest up to the triggering event). We summarize our notation:

$L,$	Barrier level or Triggering level
$T,$	Tenor or Maturity
$N,$	Total number of premium payments
$\Delta = T/N,$	Time between premium payments
$t_i = \Delta \cdot i,\ i = 1, 2, .., N,$	i^{th}periodic premium payment date
$\tilde{\zeta},$	Jump-to-Default time with intensity h
$T_L = \inf\{t : X_t = L\},$	First hitting time to trigger level L
$\zeta_L = \tilde{\zeta} \wedge T_L,$	First passage time through level L (*triggering event time*)
$\mathcal{N} = 1,$	EDS Notional
$\mathfrak{r} \in (0, 1],$	Recovery rate
$\varrho,$	EDS premium rate.

The present value of the protection payoff is given by:

$$
\begin{aligned}
PV(Protection) &= (1-\mathfrak{r}) \cdot \mathbb{E}\left[e^{-r\cdot\zeta_L}\mathbf{1}_{\{\zeta_L \le T\}}\right] \\
&= (1-\mathfrak{r}) \cdot \left(\mathbb{E}\left[e^{-r\cdot\tilde{\zeta}}\mathbf{1}_{\{\tilde{\zeta}\le T\}}\mathbf{1}_{\{T_L>\tilde{\zeta}\}}\right] + \mathbb{E}\left[e^{-r\cdot T_L}\mathbf{1}_{\{\tilde{\zeta}>T_L\}}\mathbf{1}_{\{T_L\le T\}}\right]\right) \\
&= (1-\mathfrak{r}) \cdot \Bigg(\int_0^T e^{-r\cdot u}\mathbb{E}_x\left[e^{-\int_0^u h(X_v)dv} h(X_u)\mathbf{1}_{\{T_L>u\}}\right]du \\
&\qquad + \mathbb{E}_x\left[e^{-r\cdot T_L - \int_0^{T_L} h(X_u)du}\mathbf{1}_{\{T_L \le T\}}\right]\Bigg).
\end{aligned}
\tag{2.2}
$$

The first equality follows from $\zeta_L = T_L \wedge \tilde{\zeta}$. The second equality follows from recalling that the jump-to-default time $\tilde{\zeta}$ has the intensity $h(X_t)$ and performing the standard calculations for stopping times with intensities. The first term in parenthesis is the present value of the payoff triggered by a jump-to-default from a positive stock price, if it occurs prior to maturity and prior to hitting zero via diffusion. The second term is the present value of the payoff if the stock price hits the barrier level L via diffusion, if it occurs prior to maturity and prior to the jump-to-default.

The present value of periodic premium payments up to ζ_L is given by:

$$
\begin{aligned}
PV(Premium) &= \varrho \cdot \Delta \cdot \sum_{i=1}^{N} e^{-r\cdot t_i}\mathbb{E}\left[\mathbf{1}_{\{\zeta_L \ge t_i\}}\right] \\
&= \varrho \cdot \Delta \cdot \sum_{i=1}^{N} e^{-r\cdot t_i}\mathbb{E}_x\left[e^{-\int_0^{t_i} h(X_u)du}\mathbf{1}_{\{T_L\ge t_i\}}\right].
\end{aligned}
\tag{2.3}
$$

If the triggering event occurs between the two periodic payments dates t_i and t_{i+1} (i.e., $\zeta_L \in (t_i, t_{i+1})$), the EDS buyer is required to pay the interest accrued since the previous payment date t_i up to the triggering event time ζ_L. The present value of the accrued interest is given by (here $\lfloor \cdot \rfloor$ denotes the integer part (floor) function):

$$
PV(Acc.\ Int.) = \varrho \cdot \mathbb{E}\left[e^{-r\cdot\zeta_L}\left(\zeta_L - \Delta \cdot \left\lfloor \frac{\zeta_L}{\Delta} \right\rfloor\right)\mathbf{1}_{\{\zeta_L \le T\}}\right]
$$

$$= \sum_{i=0}^{N-1} \mathbb{E}\left[e^{-r\cdot\zeta_L}\left(\zeta_L - \Delta\cdot\, i\right)\mathbf{1}_{\{\zeta_L\in(t_i,t_{i+1})\}}\right] = \mathbb{E}\left[e^{-r\cdot\zeta_L}\zeta_L\mathbf{1}_{\{\zeta_L\le T\}}\right]$$
$$-\sum_{i=1}^{N-1}(i\cdot\Delta)\cdot\left(\mathbb{E}\left[e^{-r\cdot\zeta_L}\mathbf{1}_{\{\zeta_L\le t_{i+1}\}}\right]-\mathbb{E}\left[e^{-r\cdot\zeta_L}\mathbf{1}_{\{\zeta_L\le t_i\}}\right]\right).$$

Observing that, since $\zeta_L = T_L \wedge \tilde{\zeta}$,

$$\mathbb{E}\left[e^{-r\cdot\zeta_L}\zeta_L\mathbf{1}_{\{\zeta_L\le T\}}\right] = \int_0^T u\, e^{-r\cdot u}\mathbb{E}\left[e^{-\int_0^u h(X_v)dv}h\left(X_u\right)\mathbf{1}_{\{T_L\ge u\}}\right]du$$
$$+\mathbb{E}\left[e^{-r\cdot T_L-\int_0^{T_L}h(X_u)du}T_L\mathbf{1}_{\{T_L\le T\}}\right]$$

and

$$\mathbb{E}\left[e^{-r\cdot\zeta_L}\mathbf{1}_{\{\zeta_L\le t_i\}}\right] = \int_0^{t_i} e^{-r\cdot u}\mathbb{E}\left[e^{-\int_0^u h(X_v)dv}h\left(X_u\right)\mathbf{1}_{\{T_L\ge u\}}\right]du$$
$$+\mathbb{E}\left[e^{-r\cdot T_L-\int_0^{T_L}h(X_u)du}\mathbf{1}_{\{T_L\le t_i\}}\right],$$

the present value of accrued interest can be re-written as:

$$PV(Acc.\ Int.) = \varrho\cdot\Big\{\int_0^T u\, e^{-r\cdot u}\mathbb{E}_x\left[e^{-\int_0^u h(X_v)dv}h\left(X_u\right)\mathbf{1}_{\{T_L\ge u\}}\right]du$$
$$+\mathbb{E}_x\left[e^{-r\cdot T_L-\int_0^{T_L}h(X_u)du}T_L\mathbf{1}_{\{T_L\le T\}}\right]$$
$$-\sum_{i=1}^{N-1}(i\cdot\Delta)\int_{t_i}^{t_{i+1}} e^{-r\cdot u}\mathbb{E}_x\left[e^{-\int_0^u h(X_v)dv}h\left(X_u\right)\mathbf{1}_{\{T_L\ge u\}}\right]du$$
$$-\sum_{i=1}^{N-1}(i\cdot\Delta)\Big(\mathbb{E}_x\left[e^{-r\cdot T_L-\int_0^{T_L}h(X_u)du}\mathbf{1}_{\{T_L\le t_{i+1}\}}\right]$$
$$-\mathbb{E}_x\left[e^{-r\cdot T_L-\int_0^{T_L}h(X_u)du}\mathbf{1}_{\{T_L\le t_i\}}\right]\Big)\Big\}. \tag{2.4}$$

The EDS pricing problem is to determine the arbitrage-free equity default swap rate ϱ such that it makes the present value of the swap equal to zero at inception, or, equivalently, makes the present value of the protection payment equal to the present value of the periodic premium payments and accrued interest at contract inception:

$$PV(Protection) = PV(Premium\ +\ Acc.\ Int.) \tag{2.5}$$

Observe that the expressions for the present value of the premium payments and the accrued interest are linear in the EDS rate ϱ. We can then immediately solve Eq. (2.5) for ϱ in terms of the various expectations in Eqs. (2.2), (2.3), and (2.4). In Section 3 we obtain analytical solutions for all the expectations involved, which will allow us to explicitly compute the EDS rate ϱ. The CDS swap rate is obtained in the limit $L \to 0$. In this limiting case some of the expectations vanish, and the remaining expectations simplify drastically so that they can be calculated using the results in Carr and Linetsky [11]. In contrast, the expectations with $L > 0$ involving the first hitting time T_L require us to take a different approach.

3 Solving the First Passage Time Problem for The Jump-to-Default Extended CEV Process

The instantaneous volatility in the CEV model is specified as a power function (see Cox [12], Schroder [23], Davydov and Linetsky [13,14] and Linetsky [18,19] for background on the CEV process):

$$\sigma(x) = ax^{\beta}, \tag{3.1}$$

where $\beta < 0$ is the volatility elasticity parameter and $a > 0$ is the volatility scale parameter. The CEV volatility specification exhibits the leverage effect and leads to the implied volatility skew in options prices. To be consistent with the empirical evidence linking corporate bond yields and CDS rates to equity volatility, Carr and Linetsky [11] propose to specify the default intensity as an affine function of the instantaneous stock variance:

$$h(x) = b + c\,\sigma^2(x) = b + c\,a^2 x^{2\beta}, \tag{3.2}$$

where $b \geq 0$ is a constant parameter governing the state-independent part of the jump-to-default intensity and $c \geq 0$ is a constant parameter governing the sensitivity of the intensity to the local volatility σ^2. In Carr and Linetsky [11], a and b are taken to be deterministic functions of time. In the present paper we assume that a and b are constant. The time homogeneity assumption is necessary to be able to solve the first passage time problem analytically (it is not necessary to solve the pricing problem for European options).

To value EDS contracts in the JDCEV model and, in particular, compute EDS rates, we need to calculate the following expectations:

$$\mathbb{E}_x\left[e^{-rT_L-\int_0^{T_L} h(X_u)du}\mathbf{1}_{\{T_L\leq t\}}\right],\ \mathbb{E}_x\left[e^{-rT_L-\int_0^{T_L} h(X_u)du}T_L\mathbf{1}_{\{T_L\leq t\}}\right], \tag{3.3}$$

$$\mathbb{E}_x\left[e^{-\int_0^t h(X_u)du}\mathbf{1}_{\{T_L\geq t\}}\right],\ \mathbb{E}_x\left[e^{-\int_0^t h(X_u)du}h\left(X_t\right)\mathbf{1}_{\{T_L> t\}}\right], \tag{3.4}$$

where X is the diffusion process solving the SDE (2.1) with the CEV volatility (3.1) and JDCEV default intensity (3.2) and it starts at $X_0 = x$ at time zero. Fortunately, all these expectations can be evaluated in closed form. First, we calculate the Laplace transforms of these expectations with respect to time using the analytical theory of diffusion processes. Namely, we compute in closed form the relevant resolvents. Second, we are able to invert the resulting Laplace transforms analytically. Finally, the results are substituted in the expressions for the present values of the protection payment, periodic premium payments, and the accrued interest and, after some simplifications, this yields the analytical solution to the EDS pricing problem.

To compute the first expectation in (3.3), we start by taking the Laplace transform in time:

$$\int_0^\infty e^{-sT}\mathbb{E}_x\left[e^{-rT_L-\int_0^{T_L} h(X_u)du}\mathbf{1}_{\{T_L\leq T\}}\right]dT$$
$$= \frac{1}{s}\,\mathbb{E}_x\left[e^{-(r+s)T_L-\int_0^{T_L} h(X_u)du}\right]. \tag{3.5}$$

From the analytical theory of diffusion processes, it is well known that the Laplace transform on the right hand side has the form (e.g., Borodin and Salminen [7], p.18):

$$\frac{1}{s}\,\mathbb{E}_x\left[e^{-(r+s)\cdot T_L-\int_0^{T_L} h(X_u)du}\right]=\frac{1}{s}\,\frac{\phi_{s+r}(x)}{\phi_{s+r}(L)}, \tag{3.6}$$

where $\phi_s(x)$ is the decreasing solution of the ordinary differential equation

$$\mathcal{G}u(x)=s\,u(x) \tag{3.7}$$

where $\mathcal{G}$ is the infinitesimal generator of the JDCEV diffusion process with killing at the rate h (here $\mu := r-q$):

$$\mathcal{G}u(x)=\frac{1}{2}a^2x^{2\beta+2}\frac{d^2u}{dx^2}(x)+(\mu+b+c\,a^2x^{2\beta})x\frac{du}{dx}(x)-(b+c\,a^2x^{2\beta})u(x). \tag{3.8}$$

Fortunately, in this case the ODE admits an explicit analytical solution in terms of the first and second Whittaker functions, $M_{\varkappa,m}(z)$ and $W_{\varkappa,m}(z)$, respectively.

Theorem 3.1 *For a JDCEV diffusion with the infinitesimal generator* (3.8) *with parameters $\beta<0$, $a>0$, $b\geq 0$, $c\geq 0$, and $\mu+b\neq 0$, the increasing and decreasing solutions ψ_s and ϕ_s of the ODE* (3.7) *on the interval (L,∞) and with the increasing solution satisfying the Dirichlet boundary condition at $L>0$, $\psi_s(L)=0$, are:*

$$\psi_s(x)=x^{\frac{1}{2}-c+\beta}e^{\epsilon\frac{A}{2}x^{-2\beta}}\times\Big[W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)M_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right)$$
$$-M_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)W_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\Big], \tag{3.9}$$

$$\phi_s(x)=x^{\frac{1}{2}-c+\beta}e^{\epsilon\frac{A}{2}x^{-2\beta}}W_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right), \tag{3.10}$$

where $M_{\varkappa,\nu/2}(z)$ and $W_{\varkappa,\nu/2}(z)$ are the first and second Whittaker function with indexes

$$\nu=\frac{1+2c}{2|\beta|},\;\;\varkappa(s)=\epsilon\frac{1-\nu}{2}-\frac{s+\xi}{\omega}, \tag{3.11}$$

where

$$\omega=2|\beta(\mu+b)|,\;\xi=2c(\mu+b)+b,\;A=\frac{|\mu+b|}{a^2|\beta|},\;\epsilon=\mathit{sign}(\beta(\mu+b)). \tag{3.12}$$

The Wronskian w_s of the two solutions reads:

$$w_s=\frac{2|\mu+b|\Gamma(1+\nu)}{a^2\Gamma(\nu/2+1/2-\varkappa(s))}W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right). \tag{3.13}$$

Proof. See the Appendix A.□

The Laplace transform (3.6) can be inverted analytically by applying the Cauchy Residue Theorem.

Theorem 3.2 *Let X be a JDCEV diffusion with constant parameters $\beta < 0$, $a > 0$, $b \geq 0$, $c \geq 0$ and $\mu + b \neq 0$. Then for $L > 0$ we have:*

$$\mathbb{E}_x\left[e^{-r\cdot T_L - \int_0^{T_L} h(X_u)du}\mathbf{1}_{\{T_L\leq T\}}\right]$$

$$= \left(\frac{x}{L}\right)^{\frac{1}{2}-c+\beta} e^{\epsilon\frac{A}{2}\left(x^{-2\beta}-L^{-2\beta}\right)} \times \left\{\frac{W_{\epsilon\frac{1-\nu}{2}-\frac{r+\xi}{\omega},\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{W_{\epsilon\frac{1-\nu}{2}-\frac{r+\xi}{\omega},\frac{\nu}{2}}\left(AL^{-2\beta}\right)}\right.$$

$$\left.+\sum_{n=1}^{\infty}\frac{\omega\, e^{-\left(\omega\left(\varkappa_n-\epsilon\frac{1-\nu}{2}\right)+r+\xi\right)T}}{\left(\omega\left(\varkappa_n-\epsilon\frac{1-\nu}{2}\right)+r+\xi\right)}\frac{W_{\varkappa_n,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{\left[\frac{\partial}{\partial\varkappa}W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]\Big|_{\varkappa=\varkappa_n}}\right\} \tag{3.14}$$

where

$$\{\varkappa_n, n=1,2,...\} = \left\{\varkappa \,\middle|\, W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)=0\right\}$$

are the zeros of the Whittaker function $W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)$ considered as a function of its first index with the second index and the argument kept fixed.

Proof. See the Appendix A.□

The second expectation in (3.3) immediately follows from the identity:

$$\mathbb{E}\left[e^{-rT_L-\int_0^{T_L}h(X_u)du}T_L\mathbf{1}_{\{T_L\leq T\}}\right] = -\frac{\partial}{\partial\rho}\mathbb{E}\left[e^{-\rho T_L-\int_0^{T_L}h(X_u)du}\mathbf{1}_{\{T_L\leq T\}}\right]\Big|_{\rho=r}$$

The resulting expression takes the form:

$$\mathbb{E}\left[e^{-r\cdot T_L-\int_0^{T_L}h(X_u)du}\,T_L\,\mathbf{1}_{\{T_L\leq T\}}\right]$$

$$= \left(\frac{x}{L}\right)^{\beta-c+\frac{1}{2}} e^{\epsilon\frac{A}{2}\left(x^{-2\beta}-L^{-2\beta}\right)} \times \left\{\sum_{n=1}^{\infty}\left(\frac{W_{\varkappa_n,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{\left[\frac{\partial}{\partial\varkappa}W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]\Big|_{\varkappa=\varkappa_n}}\times\right.\right.$$

$$\left.\times\;\frac{\omega\left[\left(\omega\left(\varkappa_n-\epsilon\frac{1-\nu}{2}\right)+r+\xi\right)T+1\right]e^{-\left(\omega\left(\varkappa_n-\epsilon\frac{1-\nu}{2}\right)+r+\xi\right)T}}{\left(\omega\left(\varkappa_n-\epsilon\frac{1-\nu}{2}\right)+r+\xi\right)^2}\right)+$$

$$\left.\left(\frac{\left[\frac{\partial}{\partial\rho}W_{\rho,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\right]}{\omega W_{\rho,\frac{\nu}{2}}\left(AL^{-2\beta}\right)}-\frac{W_{\rho,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\left[\frac{\partial}{\partial\rho}W_{\rho,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]}{\omega\left[W_{\rho,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]^2}\right)\Bigg|_{\rho=\frac{\epsilon(1-\nu)}{2}-\frac{r+\xi}{\omega}}\right\} \tag{3.15}$$

To compute the expectations in (3.4), we need to compute the expectations

$$\mathbb{E}_x\left[e^{-\int_0^t h(X_u)du}\mathbf{1}_{\{T_L\geq t\}}\right],\ \ \mathbb{E}_x\left[e^{-\int_0^t h(X_u)du}X_t^{2\beta}\mathbf{1}_{\{T_L>t\}}\right]. \tag{3.16}$$

From the analytical theory of diffusion processes, the Laplace transform of the expectation takes the form

$$\int_0^{\infty}e^{-st}\mathbb{E}_x\left[e^{-\int_0^t h(X_u)du}f(X_t)\mathbf{1}_{\{T_L\geq t\}}\right]dt = \int_L^{\infty}f(y)G_s(x,y)dy, \tag{3.17}$$

where $G_s(x,y)$ is the *resolvent kernel* or *Green's function* of the diffusion process X with the infinitesimal generator (3.8) and the killing boundary condition at the lower barrier L. The Green's function admits an explicit representation in terms of the increasing and decreasing solutions ψ_s and ϕ_s of the ODE (3.7) on the interval (L,∞) and with the increasing solution satisfying the Dirichlet boundary condition at L, $\psi_s(L)=0$ (see Borodin and Salminen [7], p.19; note that we define the Green's function with respect to the Lebesgue measure, while Borodin and Salminen define it with respect to the speed measure $\mathfrak{m}(y)dy$, where $\mathfrak{m}(y)$ is the speed density (A.3)):

$$G_s(x,y) = \frac{\mathfrak{m}(y)}{w_s}\begin{cases}\psi_s(x)\phi_s(y),\ x\le y\\ \psi_s(y)\phi_s(x),\ y\le x\end{cases}. \tag{3.18}$$

The *Wronskian* of the two solutions $w_s = (\psi_s'(x)\phi_s(x)-\psi_s(x)\phi_s'(x))/\mathfrak{s}(x)$ is independent of x (here $\mathfrak{s}(x)$ is the scale density (A.3) of the diffusion process). This leads to the explicit solution for the Green's function (3.18).

Theorem 3.3 *For $\mu+b\neq 0$ the Green's function of the JDCEV diffusion on the interval $(0,\infty)$ with killing at the rate h and the killing boundary at the level $L>0$ is given by:*

$$G_s(x,y) = \frac{\Gamma\left(1/2+\nu/2-\varkappa(s)\right)x^{\frac{1}{2}-c+\beta}e^{\epsilon\frac{A}{2}x^{-2\beta}}}{|\mu+b|\Gamma(1+\nu)}y^{c-\frac{3}{2}-\beta}e^{-\epsilon\frac{A}{2}y^{-2\beta}}\times$$
$$\begin{cases}\left(M_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right)-\frac{M_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)W_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)}\right)W_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right), & \underline{\underline{x\le y}}\\ \left(M_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right)-\frac{M_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)W_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right)}{W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)}\right)W_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right), & \underline{\underline{y\le x}}\end{cases}, \tag{3.19}$$

where $M_{\varkappa,m}(z)$ and $W_{\varkappa,m}(z)$ are the first and second Whittaker functions.

Proof. Substitute the fundamental solutions ψ_s and ϕ_s and the wronskian w_s from Theorem 3.1 in Eq. (3.18) (the speed density $\mathfrak{m}(x)$ of the JDCEV process is given in (A.3)). □

To compute the expectation $\mathbb{E}_x\left[e^{-\int_0^t h(X_u)du}\mathbf{1}_{\{T_L>t\}}X_t^p\right]$ (the pth moment of the process X killed at the rate h *and* at the first hitting time T_L of the lower barrier L), we first compute the integral $\int_L^\infty y^p G_s(x,y)dy$ and then invert the Laplace transform to recover the expectation. Fortunately, we are able to accomplish both in closed form. The expectations (3.4) are obtained by setting $p=0$ and $p=2\beta$, respectively.

Theorem 3.4 *Let X be a JDCEV diffusion process with the infinitesimal generator (3.8) with constant parameters $\beta<0$, $a>0$, $b\ge 0$, $c\ge 0$, and $\mu+b>0$. Then for $L>0$ and $p\in\mathbb{R}$, we have:*

$$\mathbb{E}_x[e^{-\int_0^t h(X_u)du}\mathbf{1}_{\{T_L>t\}}X_t^p]$$

$$= \sum_{n=0}^{\infty} \frac{A^{\frac{1-2c-2p}{4|\beta|}-\frac{1}{2}} \left(\frac{1-p}{2|\beta|}\right)_n \Gamma\left(1+\frac{2c+p}{2|\beta|}\right)}{n!\,\Gamma(1+\nu)} x^{\frac{1}{2}-c+\beta} \times$$

$$\times e^{-\frac{A}{2}x^{-2\beta}} e^{(p(\mu+b)-(b+\omega\, n))t} \times \left\{ M_{\frac{\nu-1}{2}+n-\left(\frac{2c+p}{2|\beta|}\right),\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\right.$$

$$\left. - \frac{M_{\frac{\nu-1}{2}+n-\left(\frac{2c+p}{2|\beta|}\right),\frac{\nu}{2}}\left(AL^{-2\beta}\right)}{W_{\frac{\nu-1}{2}+n-\left(\frac{2c+p}{2|\beta|}\right),\frac{\nu}{2}}\left(AL^{-2\beta}\right)} W_{\frac{\nu-1}{2}+n-\left(\frac{2c+p}{2|\beta|}\right),\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\right\}$$

$$+ \sum_{n=1}^{\infty} e^{-\left(\omega\left(\varkappa_n - \frac{\nu-1}{2}\right)+\xi\right)t} x^{\frac{1}{2}-c+\beta} e^{-\frac{A}{2}x^{-2\beta}} \frac{M_{\varkappa_n,\frac{\nu}{2}}\left(AL^{-2\beta}\right) W_{\varkappa_n,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{\Gamma(1+\nu)\left[\frac{d}{d\varkappa}W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]\Big|_{\varkappa=\varkappa_n}}$$

$$\times \left\{ A^{\frac{1-2c-2p}{4|\beta|}-\frac{1}{2}} \frac{\Gamma\left(1-\frac{1-p}{2|\beta|}\right)\Gamma\left(1+\frac{2c+p}{2|\beta|}\right)\Gamma\left(\frac{\nu-1}{2}-\varkappa_n-\frac{2c+p}{2|\beta|}\right)}{\Gamma\left(\frac{1-\nu}{2}-\varkappa_n\right)}\right.$$

$$- \frac{A^{\frac{1+\nu}{2}} L^{2c+p-2\beta}\Gamma(-\nu)\,\Gamma\left(\frac{1+\nu}{2}-\varkappa_n\right)}{\left(1+\frac{2c+p}{2|\beta|}\right)\Gamma\left(\frac{1-\nu}{2}-\varkappa_n\right)} {}_2F_2\left(\begin{matrix}1+\frac{2c+p}{2|\beta|},\ \frac{1+\nu}{2}-\varkappa_n \\ 2+\frac{2c+p}{2|\beta|},\ 1+\nu\end{matrix}; AL^{-2\beta}\right)$$

$$\left. - \frac{A^{\frac{1-\nu}{2}} L^{-2\beta-1+p}\Gamma(\nu)}{\left(1-\frac{1-p}{2|\beta|}\right)} {}_2F_2\left(\begin{matrix}1-\frac{1-p}{2|\beta|},\ \frac{1-\nu}{2}-\varkappa_n \\ 2-\frac{1-p}{2|\beta|},\ 1-\nu\end{matrix}; AL^{-2\beta}\right)\right\}, \tag{3.20}$$

where $\varkappa_n$ are the zeros of the Whittaker function $W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)$ as defined in Theorem 3.2, ${}_2F_2$ is the generalized hypergeometric function, $\Gamma(z)$ is the gamma function, and $(z)_n = \Gamma(z+n)/\Gamma(z)$ is the Pochhammer symbol.

For $L = 0$ and $p > 2(\beta - c)$ we have:

$$\mathbb{E}_x[e^{-\int_0^t h(X_u)du}\mathbf{1}_{\{T_L>t\}}X_t^p] = \frac{A^{\frac{1-2c-2p}{4|\beta|}-\frac{1}{2}}\Gamma\left(1+\frac{2c+p}{2|\beta|}\right) x^{\frac{1}{2}-c+\beta}}{e^{\frac{A}{2}x^{-2\beta}}\Gamma(1+\nu)} \times$$

$$\times \sum_{n=0}^{\infty} e^{(p(\mu+b)-(b+\omega\, n))t} \frac{\left(\frac{1-p}{2|\beta|}\right)_n}{n!} M_{\frac{\nu-1}{2}+n-\left(\frac{2c+p}{2|\beta|}\right),\frac{\nu}{2}}\left(Ax^{-2\beta}\right). \tag{3.21}$$

Proof. See the Appendix A.□

Remark 3.1. To save space, we formulated Theorem 3.4 for the case $\mu + b > 0$. The expressions for the case $\mu + b < 0$ are similar and are included in Appendix B.

4 Pricing EDS under the JDCEV Model

Having explicitly computed all the required expectations, we are now ready to write down the results for the present values of the protection payoff, periodic premium payments, and accrued interest in the JDCEV model. Expressions (2.2) and (2.4) for the protection payment and the accrued interest involve integrals with respect to time. These integrals are computed in closed form. In addition, the expressions for the accrued interest and the periodic premium payments contain summations over the coupon payment dates t_i (see Eqs. (2.4) and (2.3)). These sums collapse to closed form expressions by means of identities $\sum_{i=1}^{N} a^i = a\left(1-a^N\right)/(1-a)$, $\sum_{i=1}^{N} i\cdot a^i = (a-(1+N)\,a^{N+1}+Na^{N+2})/((a-1)^2)$. The final results for the present values of the protection payment, periodic premium payments, and accrued interest simplify as follows.

4.1 Protection Payoff

The present value of the EDS protection payoff (2.2) with the triggering barrier $L>0$ reduces to the following expression in the JDCEV model with $\mu+b>0$ (similar expressions for $\mu+b<0$ can be obtained by using the result in Appendix B):

$$
\begin{aligned}
PV(Protection) &= (1-\mathfrak{r})\cdot\left(\frac{x^{\frac{1}{2}-c+\beta}\Gamma\left(\frac{c}{|\beta|}+1\right)}{\Gamma(1+\nu)}e^{-\frac{A}{2}x^{-2\beta}}\right.\\
&\times\sum_{n=1}^{\infty}\left\{\frac{b\,\mathcal{L}_+(n-1,0)}{(r+b+\omega\,(n-1))}\left(1-e^{-(r+b+\omega\,(n-1))T}\right)\right.\\
&\quad+\frac{|\beta|a^2\,\mathcal{L}_+(n-1,2\beta)}{(r+b+\omega\,n)}\left(1-e^{-(r+b+\omega\,n)T}\right)\\
&\left.\quad+\frac{b\,\mathcal{M}_+(n,0)+|\beta|a^2\mathcal{M}_+(n,2\beta)}{\left(r+\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+\xi\right)}\left(1-e^{-\left(r+\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+\xi\right)T}\right)\right\}\\
&\quad+\left(\frac{x}{L}\right)^{\beta-c+\frac{1}{2}}e^{-\frac{A}{2}\left(x^{-2\beta}-L^{-2\beta}\right)}\left\{\frac{W_{\frac{\nu-1}{2}-\frac{r+\xi}{\omega},\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{W_{\frac{\nu-1}{2}-\frac{r+\xi}{\omega},\frac{\nu}{2}}\left(AL^{-2\beta}\right)}\right.\\
&\left.\left.+\sum_{n=1}^{\infty}\frac{\omega\,e^{-\left(\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+r+\xi\right)T}}{\left(\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+r+\xi\right)}\frac{W_{\varkappa_n,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{\left[\frac{\partial}{\partial\varkappa}W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]\Big|_{\varkappa=\varkappa_n}}\right\}\right).
\end{aligned}
\tag{4.1}
$$

In the CDS limit $L=0$, the expression simplifies to:

$$
\begin{aligned}
PV(Protection) &= (1-\mathfrak{r})\cdot\frac{x^{\frac{1}{2}-c+\beta}\Gamma\left(\frac{c}{|\beta|}+1\right)}{\Gamma(1+\nu)}e^{-\frac{A}{2}x^{-2\beta}}\\
&\times\sum_{n=1}^{\infty}\left\{\frac{b\,\mathcal{D}_+(n-1,0)\left(1-e^{-(r+b+\omega\,(n-1))T}\right)}{(r+b+\omega\,(n-1))}\right.
\end{aligned}
$$

$$+\frac{|\beta| a^2\,\mathcal{D}_+(n-1,2\beta)\left(1-e^{-(r+b+\omega\, n)T}\right)}{(r+b+\omega\, n)}\Bigg\}. \tag{4.2}$$

In Eqs. (4.1) and (4.2) we have introduced the following notation:

$$\begin{aligned}\mathcal{L}_+(n,p) := \frac{A^{\frac{1-2c}{4|\beta|}+\frac{1}{2}-\delta_p}\left(1+\frac{1}{2|\beta|}-\delta_p\right)_n}{n!} \times \Bigg\{ M_{\frac{1-2c}{4|\beta|}+\frac{2n+1}{2}-\delta_p,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\\ -\frac{M_{\frac{1-2c}{4|\beta|}+\frac{2n+1}{2}-\delta_p,\frac{\nu}{2}}\left(AL^{-2\beta}\right)}{W_{\frac{1-2c}{4|\beta|}+\frac{2n+1}{2}-\delta_p,\frac{\nu}{2}}\left(AL^{-2\beta}\right)} W_{\frac{1-2c}{4|\beta|}+\frac{2n+1}{2}-\delta_p,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\Bigg\},\end{aligned} \tag{4.3}$$

$$\begin{aligned}\mathcal{M}_+(n,p) := &\frac{M_{\varkappa_n,\frac{\nu}{2}}\left(AL^{-2\beta}\right)W_{\varkappa_n,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{\left[\frac{d}{d\varkappa}W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]\Big|_{\varkappa=\varkappa_n}}\\ &\times\left\{\frac{\Gamma(\nu)\,{}_2F_2\left(\begin{matrix}\delta_p-\frac{1}{2|\beta|}, & \frac{1-\nu-2\varkappa_n}{2}\\ 1+\delta_p-\frac{1}{2|\beta|}, & 1-\nu\end{matrix};AL^{-2\beta}\right)}{A^{\frac{\nu-1}{2}}L^{2\beta\delta_p+1}\Gamma\left(\frac{c}{|\beta|}+\delta_p\right)\left(\frac{1}{2|\beta|}-\delta_p\right)}\right.\\ &+A^{\frac{1-2c}{4|\beta|}+\frac{1}{2}-\delta_p}\frac{\Gamma\left(\delta_p-\frac{1}{2|\beta|}\right)\Gamma\left(\frac{1-2c}{4|\beta|}+\frac{1}{2}-\delta_p-\varkappa_n\right)}{\Gamma\left(\frac{1-\nu}{2}-\varkappa_n\right)}\\ &\left.-\frac{\Gamma(-\nu)\Gamma\left(\frac{1+\nu-2\varkappa_n}{2}\right)\,{}_2F_2\left(\begin{matrix}\delta_p+\frac{c}{|\beta|}, & \frac{1+\nu-2\varkappa_n}{2}\\ 1+\delta_p+\frac{c}{|\beta|}, & 1+\nu\end{matrix};AL^{-2\beta}\right)}{A^{-\frac{1+\nu}{2}}L^{2\beta\delta_p-2c}\Gamma\left(\frac{c}{|\beta|}+\delta_p+1\right)\Gamma\left(\frac{1-\nu}{2}-\varkappa_n\right)}\right\},\end{aligned} \tag{4.4}$$

$$\mathcal{D}_+(n,p) := A^{\frac{1-2c}{4|\beta|}+\frac{1}{2}-\delta_p}\frac{\left(1+\frac{1}{2|\beta|}-\delta_p\right)_n}{n!}M_{\frac{1-2c}{4|\beta|}+\frac{2n+1}{2}-\delta_p,\frac{\nu}{2}}\left(Ax^{-2\beta}\right), \tag{4.5}$$

where $\{\varkappa_n, n=1,2,\ldots\} = \left\{\varkappa\middle|\, W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)=0\right\}$ and $\delta_p = \begin{cases}1, p=0\\ 0, p=2\beta\end{cases}.$

4.2 Periodic Premium Payments

The present value of the EDS periodic premium payments (2.3) with the triggering barrier $L>0$ reduces to the following expression in the JDCEV model with $\mu+b>0$:

$$\begin{aligned}PV(Premium) = \varrho\cdot\Delta\cdot\frac{x^{\frac{1}{2}-c+\beta}\Gamma\left(\frac{c}{|\beta|}+1\right)}{\Gamma(1+\nu)}e^{-\frac{A}{2}x^{-2\beta}}\\ \times\sum_{n=1}^{\infty}\Bigg\{\left(\frac{1-e^{-(r+b+\omega\,(n-1))\Delta_t N}}{e^{(r+b+\omega\,(n-1))\Delta_t}-1}\right)\mathcal{L}_+(n-1,0)\\ +\left(\frac{1-e^{-\left(r+\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+\xi\right)\Delta_t N}}{e^{\left(r+\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+\xi\right)\Delta_t}-1}\right)\mathcal{M}_+(n,0)\Bigg\}.\end{aligned} \tag{4.6}$$

In the CDS limit $L = 0$ the expression simplifies to:

$$PV(Premium) = \varrho \cdot \Delta \cdot \frac{x^{\frac{1}{2}-c+\beta}\Gamma\left(\frac{c}{|\beta|}+1\right)}{\Gamma(1+\nu)} e^{-\frac{A}{2}x^{-2\beta}} \times$$
$$\times \sum_{n=1}^{\infty} \left(\frac{1 - e^{-(r+b+\omega\,(n-1))\Delta_t N}}{e^{(r+b+\omega\,(n-1))\Delta_t} - 1} \right) \mathcal{D}_+(n-1,0) \tag{4.7}$$

4.3 Accrued Interest

The present value of the EDS accrued interest payment (2.4) with the triggering barrier $L > 0$ reduces to the following expression in the JDCEV model with $\mu + b > 0$:

$$PV(Acc.\ Int.) = \varrho \frac{x^{\frac{1}{2}-c+\beta} e^{-\frac{A}{2}x^{-2\beta}} \Gamma\left(\frac{c}{|\beta|}+1\right)}{\Gamma(1+\nu)}$$
$$\times \sum_{n=1}^{\infty} \left\{ \frac{b\,\mathcal{L}_+(n-1,0)\left(1-e^{-(r+b+\omega\,(n-1))T}\right)}{(r+b+\omega\,(n-1))} \times \right.$$
$$\times \left(\frac{1}{(r+b+\omega\,(n-1))} + \frac{\Delta}{\left(1-e^{(r+b+\omega\,(n-1))\Delta}\right)} \right)$$
$$+ \frac{\mathcal{L}_+(n-1,2\beta)\left(1-e^{-(r+b+\omega\,n)T}\right)}{(r+b+\omega\,n)\,/(|\beta|a^2)} \left(\frac{1}{(r+b+\omega\,n)} + \frac{\Delta}{\left(1-e^{(r+b+\omega\,n)\Delta}\right)} \right)$$
$$+ \frac{\left(b\,\mathcal{M}_+(n,0) + |\beta|a^2\,\mathcal{M}_+(n,2\beta)\right)\left(1-e^{-\left(r+\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+\xi\right)T}\right)}{\left(r+\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+\xi\right)} \times$$
$$\left. \times \left(\frac{1}{\left(r+\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+\xi\right)} + \frac{\Delta}{\left(1-e^{\left(r+\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+\xi\right)\Delta}\right)} \right) \right\}$$
$$+\varrho \left(\frac{x}{L}\right)^{\beta-c+\frac{1}{2}} \frac{e^{-\frac{Ax^{-2\beta}}{2}}}{e^{-\frac{AL^{-2\beta}}{2}}} \left\{ \sum_{n=1}^{\infty} \frac{W_{\varkappa_n,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{\left(\frac{(2\varkappa_n-\nu+1)}{2}+\frac{r+\xi}{\omega}\right)\left[\frac{\partial}{\partial\varkappa}W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]\Big|_{\varkappa=\varkappa_n}} \right.$$
$$\times \left(\frac{e^{-\left(\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+r+\xi\right)T}}{\left(\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+r+\xi\right)} - \frac{\Delta\left(1-e^{-\left(\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+r+\xi\right)T}\right)}{\left(1-e^{\left(\omega\left(\varkappa_n-\frac{\nu-1}{2}\right)+r+\xi\right)\Delta_t}\right)} \right)$$
$$\left. + \left(\frac{\left[\frac{\partial}{\partial\rho}W_{\rho,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\right]}{\omega\left[W_{\rho,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]} - \frac{W_{\rho,\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\left[\frac{\partial}{\partial\rho}W_{\rho,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]}{\omega\left[W_{\rho,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]^2} \right) \Bigg|_{\rho=\frac{\nu-1}{2}-\frac{r+\xi}{\omega}} \right\} \tag{4.8}$$

In the CDS limit $L = 0$ the expression simplifies to:

$$PV(Acc.\ Int.) = \varrho \cdot \frac{x^{\frac{1}{2}-c+\beta}\Gamma\left(\frac{c}{|\beta|}+1\right)}{\Gamma(1+\nu)} e^{-\frac{A}{2}x^{-2\beta}} \times$$
$$\times \sum_{n=1}^{\infty} \left\{ \frac{b\,\mathcal{D}_+(n-1,0)\left(1-e^{-(r+b+\omega\,(n-1))T}\right)}{(r+b+\omega\ (n-1))} \times \right.$$
$$\times \left(\frac{1}{(r+b+\omega\ (n-1))} + \frac{\Delta}{\left(1-e^{(r+b+\omega\,(n-1))\Delta}\right)} \right)$$
$$\left. + \frac{\mathcal{D}_+(n-1,2\beta)\left(1-e^{-(r+b+\omega\,n)T}\right)}{(r+b+\omega\,n)\,/(|\beta|a^2)} \left(\frac{1}{(r+b+\omega\,n)} + \frac{\Delta}{\left(1-e^{(r+b+\omega\,n)\Delta}\right)} \right) \right\} \tag{4.9}$$

5 Numerical Examples

To illustrate the results in section 4, in this section we consider EDS contracts written on the JDCEV process with the parameters listed in Table 5.1. The risk-free rate r is 5%, the dividend yield q is zero, the volatility elasticity parameter is $\beta = -1$, the initial stock price is $S_0 = \$50$. We consider two values of the volatility scale parameter a to illustrate the impact of the at-the-money volatility level. We choose a in the local volatility function $\sigma(S) = aS^{\beta}$ so that when the stock price is fifty dollars ($S = 50$), the local volatility at this level $\sigma(50)$ is equal to 20% per annum in our first example (Example 1) and 40% in the second example (Example 2), namely $a = 0.20 * 50^{-1} = 10$ and $a = 0.40 * 50^{-1} = 20$, respectively. To compare the standard CEV model and the JDCEV model, we consider both the standard CEV and the JDCEV specifications. In the standard CEV model $b = c = 0$. In the JDCEV model the constant part of the default intensity b is taken to be 2% per annum. To illustrate the effect of increasing the influence of the volatility on the default intensity, we consider two specifications $c = 1$ and $c = 2$ in the JDCEV case. We consider CDS and EDS contracts with the recovery rate $\mathfrak{r}$ of 50% per dollar of notional and with quarterly premium payments, i.e., $\Delta = 0.25$ years. We consider EDS contracts with triggering levels at 30% and 50% of the initial stock price. We denote these contracts EDS30 and EDS50, respectively.

Figure 5.1 plots the CDS, EDS30 and EDS50 swap rates as functions of the swap maturity (tenor) for the cases with $\sigma(50) = 0.2$. We observe that the behavior of the EDS and CDS rate curves is strikingly different in the standard CEV and JDCEV models. In the standard CEV model, the curves start at zero, as there is no jump to default. In the JDCEV model, the curves start at one half the instantaneous jump-to-default intensity (recall that the recovery rate is set to fifty percent of the notional amount), as CDS and EDS contracts may all be simultaneously triggered by a jump to default. We also observe that in the CEV model there are large spreads between

Parameters	Example 1	Example 2
Δ	0.25	0.25
$\mathfrak{r}$	0.5	0.5
L	$\{0, 15, 25\}$	$\{0, 15, 25\}$
S_0	50	50
r	0.05	0.05
q	0	0
(b, c) pairs	{(0,0), (0.02,1), (0.02,2)}	{(0,0), (0.02,1)}
β	-1	-1
σ	0.20	0.40

Table 5.1 *CDS and EDS specification and JDCEV parameter values.*

the CDS, EDS30, and EDS50 rates. This is due to the fact that the contracts can only be triggered by the CEV diffusion hitting lower barrier levels at $0.5 \times S_0$, $0.3 \times S_0$, and at zero. In the JDCEV model the corresponding spreads are much smaller since, in addition to hitting the levels via continuous diffusion, all the contracts may be simultaneously triggered by a jump to default. In particular, CDS and EDS30 rates are very close, indicating that for this set of parameter specifications EDS30 contracts behave similar to CDS contracts. The rates on EDS50 contracts are significantly higher, since the probability of hitting the level equal to fifty percent of the initial asset price via continuous diffusion is substantial, in addition to the probability of a jump to default that is common to all the contracts. We also observe that the spreads of the EDS contracts over the CDS contracts decrease as the default intensity parameter c increases. This is due to the increased probability of a jump to default triggering all the contracts simultaneously, as well as the decrease in the probability that no jump to default occurs prior to maturity (CDS is not triggered), but the stock price falls to the triggering barrier level (EDS is triggered). These observations indicate that the JDCEV model specification is in better agreement with the empirically observed high credit and equity event correlation (Jobst and de Servigny [16] Kendall's tau) than the alternative models employed in the literature for the valuation of EDS that do not include the possibility of a jump to default, such as the standard CEV or Lévy process based models.

Figure 5.2 plots the CDS, EDS30 and EDS50 swap rates as functions of the swap maturity (tenor) for the cases with higher volatility $\sigma(50) = 0.4$. Again, the standard CEV swap rate curves start at zero, but increase rapidly due to increased probability of hitting the lower barriers via diffusion in the model with higher volatility. The JDCEV curves start at one half the default intensity. The spreads between CDS and EDS contracts are much smaller in the JDCEV model that in the CEV model due to the substantial probability of a jump to default triggering all the contracts simultaneosly. However, in this higher volatility situation in the JDCEV model the spreads of EDS rates over CDS rates are greater than the very tight spreads observed in the lower volatility situation in Figure 1. This is due to the fact that, while the jump to default probability increases with the increase in the volatility through the default intensity dependence on volatility, the probability of hitting the lower barriers by continuous diffusion increases as well. To further illustrate, Tables 6.1-6.2 show each

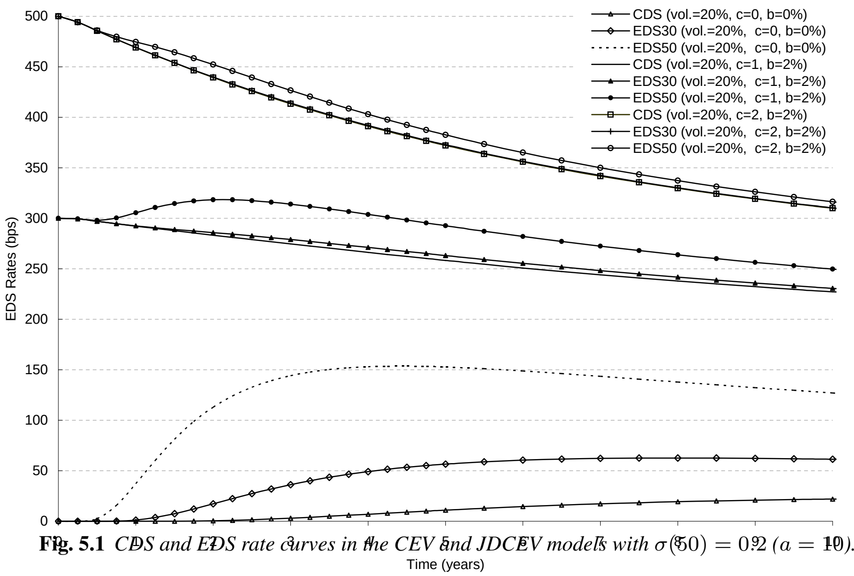

Fig. 5.1 *CDS and EDS rate curves in the CEV and JDCEV models with* $\sigma(50) = 0.2$ *(*$a = 10$*).*

of the components needed to price the EDS and CDS contracts, i.e., the present value of the protection payoff, premium payments, and accrued interest, under the CEV and JDCEV models.[2] We observe that the present value of the protection payment under the CEV model is very small for short maturities. The comparison of the swap curves under the standard CEV and JDCEV models in the higher volatility scenario further demonstrates that the JDCEV model offers a significant improvement in financial realism over the standard CEV model for valuing CDS and EDS contracts. The standard CEV model produces CDS and EDS swap rates that are unrealistically low for short maturities, then increase rapidly with maturity, and produce an exaggerated hump for mid-maturities. Moreover, the EDS over CDS spreads produced under the CEV specification with high volatility are much larger than those produced under the JDCEV. This is in contradiction with the empirical evidence produced by Jobst and de Servigny [16].

6 Conclusions

Equity default swaps (EDS) are hybrid credit-equity products that provide a bridge from credit default swaps (CDS) to equity derivatives with barriers. In this paper we

[2] In our implementation we used the built-in functions in *Mathematica* for the evaluation of Laguerre and generalized hypergeometric functions ${}_2F_2$ functions. To compute the Whittaker functions, their derivatives with respect to the first index, and their roots with respect to the first index $\varkappa_n$, we implemented the algorithms developed by Abad and Sesma [1,2]. We found that these algorithms converge faster than the built-in confluent hypergeometric functions in *Mathematica* for large values of the first index that are available as built-in functions in *Mathematica*.

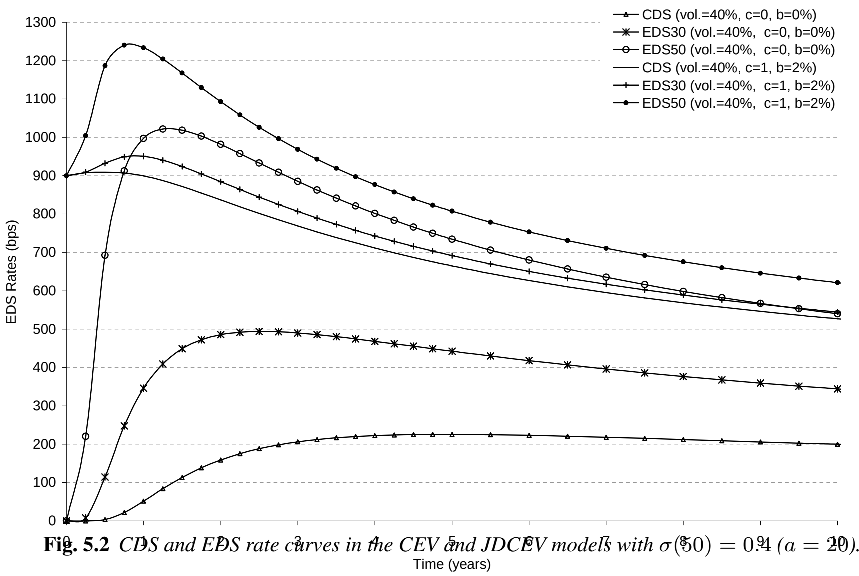

Fig. 5.2 *CDS and EDS rate curves in the CEV and JDCEV models with $\sigma(50) = 0.4$ ($a = 20$).*

develop an analytical solution to the EDS pricing problem under the Jump-to-Default Extended Constant Elasticity Variance Model of Carr and Linetsky [11]. We argue that the JDCEV model is well suited to study such hybrid credit-equity products as EDS, as it naturally incorporates the empirically observed relationships between the stock price, stock price volatility, and credit spreads. Mathematically, we obtain an analytical solution to the first passage time problem for the JDCEV diffusion process with killing. In particular, we obtain analytical results for the present values of the protection payoff at the triggering event, periodic premium payments up to the triggering event, and the interest accrued from the previous periodic premium payment up to the triggering event, and determine arbitrage-free equity default swap rates and compare them with CDS rates. Generally, the EDS rate is strictly greater than the corresponding CDS rate. However, when the triggering barrier is set to be a low percentage of the initial stock price and the volatility of the underlying firm's stock price is moderate, the EDS rates and CDS rates are quite close. Given the current movement to list CDS contracts on organized derivatives exchanges to alleviate the problems with the counterparty risk and the opacity of over-the-counter CDS trading, we argue that EDS contracts may prove to be an interesting alternative to CDS contracts offering some advantages due to the unambiguity and transparency of the triggering event based on the observable stock price.

Time (yrs)	Prot.	Prem.	Acc. Interest	CDS (bps)	Prot.	Prem.	Acc. Interest	EDS30 (bps)	Prot.	Prem.	Acc. Interest	EDS50 (bps)
0.25	0.0000	0.2469	0.0000	0	0.0002	0.2468	0.0001	8	0.0054	0.2442	0.0022	221
0.5	0.0002	0.4906	0.0001	3	0.0056	0.4878	0.0018	114	0.0333	0.4715	0.0097	693
1	0.0050	0.9660	0.0014	52	0.0329	0.9414	0.0088	346	0.0896	0.8748	0.0234	997
2	0.0295	1.8497	0.0077	159	0.0852	1.7335	0.0217	485	0.1550	1.5392	0.0392	982
3	0.0547	2.6397	0.0139	206	0.1194	2.4078	0.0301	490	0.1890	2.0866	0.0475	885
5	0.0905	3.9880	0.0228	226	0.1577	3.5237	0.0395	443	0.2228	2.9778	0.0558	734
7	0.1119	5.1055	0.0281	218	0.1773	4.4332	0.0444	396	0.2389	3.6994	0.0598	636
10	0.1300	6.4778	0.0325	200	0.1925	5.5443	0.0481	344	0.2508	4.5804	0.0627	540

Table 6.1 *Present values of the protection payment, periodic premium payments, and the accrued interest rate payment per one dollar of notional amount, and the corresponding annualized CDS and EDS premium rates ρ(in basis points per annum) for the standard CEV model ($b = c = 0$) with $\sigma(50) = 40\%$ and $\beta = -1$.*

Time (yrs)	Prot.	Prem.	Acc. Interest	CDS (bps)	Prot.	Prem.	Acc. Interest	EDS30 (bps)	Prot.	Prem.	Acc. Interest	EDS50 (bps)
0.25	0.0219	0.2360	0.0054	908	0.0220	0.2360	0.0055	910	0.0242	0.2349	0.0064	1004
0.5	0.0427	0.4588	0.0106	909	0.0437	0.4582	0.0109	932	0.0552	0.4514	0.0142	1187
1	0.0799	0.8680	0.0197	900	0.0841	0.8640	0.0208	950	0.1065	0.8371	0.0266	1233
2	0.1341	1.5689	0.0330	837	0.1405	1.5536	0.0346	884	0.1661	1.4786	0.0410	1093
3	0.1689	2.1553	0.0416	769	0.1752	2.1282	0.0431	807	0.1993	2.0086	0.0492	969
5	0.2097	3.1019	0.0516	665	0.2150	3.0554	0.0530	692	0.2359	2.8628	0.0582	808
7	0.2324	3.8472	0.0572	595	0.2371	3.7861	0.0584	617	0.2558	3.5375	0.0631	711
10	0.2521	4.7191	0.0621	527	0.2563	4.6418	0.0632	545	0.2732	4.3297	0.0674	621

Table 6.2 *Present values of the protection payment, periodic premium payments, and the accrued interest rate payment per one dollar of notional amount, and the corresponding annualized CDS and EDS premium rates ρ(in basis points per annum) for the JDCEV model with $\sigma(50) = 40\%$, $b = 2\%$, and $c = 1$.*

A Proofs of Theorems

A.1 Proof of Theorem 3.1

Consider the ODE (3.7) with $\mu + b \neq 0$ and the transformation $u(x) = x^{\frac{1}{2}-c+\beta} e^{\epsilon \frac{A}{2} x^{-2\beta}} v(y)$ where $y = Ax^{-2\beta}$. The ODE reduces to the Whittaker equation for the function $v(y)$:

$$\frac{d^2 v}{dy^2}(y) + \left(-\frac{1}{4} + \frac{\varkappa(s)}{y} + \frac{1-\nu^2}{4y^2} \right) v(y) = 0, \tag{A.1}$$

with $A, \epsilon, \nu, \varkappa(s), \xi$, and ω defined in (3.12). Inverting the change of variables, the increasing and decreasing solutions of the Whittaker ODE (A.1) are given by the Whittaker functions $v_1(y) = M_{\varkappa(s), \frac{\nu}{2}}(y)$ and $v_2(y) = W_{\varkappa(s), \frac{\nu}{2}}(y)$, respectively. The Wronskian is given by:

$$\mathfrak{W}(v_1, v_2)(y) := v_1(y) v_2'(y) - v_1'(y) v_2(y) = -\frac{\Gamma(1+\nu)}{\Gamma\left(\frac{1+\nu}{2} - \varkappa(s)\right)}. \tag{A.2}$$

For $\beta < 0$, the increasing solution of the ODE (3.7) on the interval (L, ∞) satisfying the boundary condition $u(L) = 0$ is:

$$\psi_s(x) = x^{\frac{1}{2}-c+\beta} e^{\epsilon \frac{A}{2} x^{-2\beta}} \Big[W_{\varkappa(s), \frac{\nu}{2}}\left(AL^{-2\beta}\right) M_{\varkappa(s), \frac{\nu}{2}}\left(Ax^{-2\beta}\right)$$
$$- M_{\varkappa(s), \frac{\nu}{2}}\left(AL^{-2\beta}\right) W_{\varkappa(s), \frac{\nu}{2}}\left(Ax^{-2\beta}\right) \Big].$$

The decreasing solution vanishing at infinity, $\lim_{x\uparrow\infty} u(x) = 0$, is:

$$\phi_s(x) = x^{\frac{1}{2}-c+\beta} e^{\epsilon \frac{A}{2} x^{-2\beta}} W_{\varkappa(s), \frac{\nu}{2}}\left(Ax^{-2\beta}\right).$$

The Wronskian is:

$$\mathfrak{W}(\phi_s, \psi_s)(x) = 2|\beta| A x^{-2c} e^{\epsilon\, A x^{-2\beta}} \frac{\Gamma(1+\nu)}{\Gamma\left(\frac{1+\nu}{2} - \varkappa(s)\right)} W_{\varkappa(s), \frac{\nu}{2}}\left(AL^{-2\beta}\right).$$

The speed and scale densities of the JDCEV diffusion are:

$$\mathfrak{m}(x) = \frac{2}{a^2} x^{2c-2-2\beta} e^{-\epsilon\, A x^{-2\beta}}, \quad \mathfrak{s}(x) = x^{-2c} e^{\epsilon\, A x^{-2\beta}}. \tag{A.3}$$

The Wronskian with respect to the scale density $w_s := \mathfrak{W}(\phi_s(x), \psi_s(x)) / \mathfrak{s}(x)$ is obtained by dividing the Wronskian by the scale density.□

A.2 Proof of Theorem 3.2

The Laplace transform in Eq. (3.5), where we substitute the result (3.10) in the explicit representation for the Laplace transform (3.6), can be inverted via the Bromwich Laplace inversion formula:

$$\mathbb{E}_x \left[e^{-r \cdot T_L - \int_0^{T_L} h(X_u) du} \mathbf{1}_{\{T_L \leq T\}} \right]$$
$$= \left(\frac{x}{L}\right)^{\frac{1}{2}-c+\beta} e^{\epsilon \frac{A}{2}\left(x^{-2\beta} - L^{-2\beta}\right)} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{sT} \frac{W_{\varkappa(s+r), \frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{s\, W_{\varkappa(s+r), \frac{\nu}{2}}\left(AL^{-2\beta}\right)} \frac{ds}{2\pi i}.$$

As a function of the complex variable s, the integrand has simple poles at $s = 0$ and at $s = s_n = -\omega\left(\varkappa_n - \epsilon \frac{1-\nu}{2}\right) - (r + \xi)$, where $\{\varkappa_n, n = 1, 2, \ldots\} = \left\{ \varkappa \,\middle|\, W_{\varkappa, \frac{\nu}{2}}\left(AL^{-2\beta}\right) = 0 \right\}$. The zeros

$\varkappa_n$ of the function $W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)$ are located on the real line with $\varkappa > \frac{1+\nu}{2}$ (see Slater [24] page 105), which implies that the poles of the integrand are located on the real line (s real) with

$$s < -\omega\left(\frac{1+\nu}{2} - \epsilon\frac{1-\nu}{2}\right) - (r+\xi) \Rightarrow \begin{cases} s < -(\omega\nu + r + \xi) < 0, \ \epsilon = 1 \\ s < -(\omega + r + \xi) < 0, \quad \epsilon = -1 \end{cases}.$$

Applying the Cauchy Residue Theorem and calculating the residues at $s = 0$ and $s = s_n = -\omega\big(\varkappa_n - \epsilon\frac{1-\nu}{2}\big) - (r+\xi)$, we arrive at the result (3.14).□

A.3 Proof of Theorem 3.4

We start with the representation:

$$\mathbb{E}_x\left[e^{-\int_0^t h(X_u)du} f(X_t)\mathbf{1}_{\{T_L > t\}}\right] = \frac{1}{2\pi\, i}\int_{\gamma - i\,\infty}^{\gamma + i\,\infty} e^{st}\mathcal{R}_s f(x) ds, \tag{A.4}$$

where $\mathcal{R}_s$ is the resolvent operator of the JDCEV diffusion process on the interval (L,∞) killed at the rate h and killed at the lower boundary L. The resolvent operator can be written as the integral operator $\mathcal{R}_s f(x) = \int_L^\infty f(y)G_s(x,y)dy$ with the resolvent kernel (Green's function) (3.19), assuming the integrability condition $\int_L^\infty |f(y)G_s(x,y)|dy < \infty$ is satisfied. We first consider the case with $L > 0$ (EDS). Using the explicit expression for the Green's function $G_s(x,y)$ (3.19) with $\epsilon = -1$ (for $\beta < 0$ and $\mu + b > 0$), for the power function $f(x) = x^p$ we obtain:

$$\begin{aligned}
\mathcal{R}_s x^p = \frac{\Gamma\left(\frac{1+\nu}{2} - \varkappa(s)\right)}{(\mu+b)\Gamma(1+\nu)} x^{\frac{1}{2}-c+\beta} e^{-\frac{A}{2}x^{-2\beta}} \\
\times\Bigg\{\Bigg[M_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right) - \frac{M_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)}{W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)} W_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\Bigg] \times \\
\times \int_x^\infty y^{p+c-\frac{3}{2}-\beta} e^{\frac{A}{2}y^{-2\beta}} W_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right) dy \\
+ W_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\int_L^x y^{p+c-\frac{3}{2}-\beta} e^{\frac{A}{2}y^{-2\beta}}\Big[M_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right) \\
- \frac{M_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)}{W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)} W_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right)\Big]dy\Bigg\}.
\end{aligned} \tag{A.5}$$

Using the identity $W_{k,m}(x) = \frac{\Gamma(-2m)}{\Gamma(1/2-m-k)}M_{k,m}(x) - \frac{\Gamma(-2m)\Gamma(1+2m)}{\Gamma(1/2+m-k)\Gamma(1-2m)}M_{k,-m}(x)$, we further rewrite the resolvent as:

$$\begin{aligned}
\mathcal{R}_s x^p = \frac{\Gamma\left(\frac{1+\nu}{2} - \varkappa(s)\right)}{(\mu+b)\Gamma(1+\nu)} x^{\frac{1}{2}-c+\beta} e^{-\frac{A}{2}x^{-2\beta}}\Big\{M_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right) \\
\times\Big[\int_x^\infty y^{p+c-\frac{3}{2}-\beta} e^{\frac{A}{2}y^{-2\beta}} W_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right) dy \\
+ \frac{\Gamma(-\nu)}{\Gamma(\frac{1-\nu}{2} - \varkappa(s))}\int_L^x y^{p+c-\frac{3}{2}-\beta} e^{\frac{A}{2}y^{-2\beta}} M_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right) dy\Big] \\
- \frac{M_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right) W_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)}\int_L^\infty y^{p+c-\frac{3}{2}-\beta} e^{\frac{A}{2}y^{-2\beta}} W_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right) dy \\
- \frac{\Gamma(-\nu)\,\Gamma(1+\nu)\, M_{\varkappa(s),-\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{\Gamma(\frac{1+\nu}{2} - \varkappa(s))\Gamma(1-\nu)}\int_L^x y^{p+c-\frac{3}{2}-\beta} e^{\frac{A}{2}y^{-2\beta}} M_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right) dy\Big\}.
\end{aligned} \tag{A.6}$$

The integrals are calculated in closed form by using the integrals for the Whittaker functions in Prudnikov et al. [21] pp.39-40:

$$\int_x^\infty y^{p+c-\frac{3}{2}-\beta} e^{\frac{A}{2}y^{-2\beta}} W_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right)dy$$
$$= \frac{A^{\frac{1-2c-2p}{4|\beta|}-\frac{1}{2}}}{2|\beta|}\frac{\Gamma\left(1-\frac{1-p}{2|\beta|}\right)\Gamma\left(1+\frac{2c+p}{2|\beta|}\right)\Gamma\left(\frac{s+\xi}{\omega}-\frac{2c+p}{2|\beta|}\right)}{\Gamma\left(\frac{s+\xi}{\omega}+1\right)\Gamma\left(\frac{s+\xi}{\omega}+1-\nu\right)}$$
$$-\frac{A^{\frac{1-\nu}{2}}x^{-2\beta-1+p}\Gamma(\nu)}{(2|\beta|-1+p)\Gamma\left(\frac{s+\xi}{\omega}+1\right)}\,{}_2F_2\left(\begin{matrix}1-\frac{1-p}{2|\beta|},\ 1+\frac{s+\xi}{\omega}-\nu\\ 2-\frac{1-p}{2|\beta|},\ 1-\nu\end{matrix};Ax^{-2\beta}\right)$$
$$-\frac{A^{\frac{1+\nu}{2}}x^{2c+p-2\beta}\Gamma(-\nu)}{(2|\beta|+2c+p)\Gamma\left(\frac{s+\xi}{\omega}+1-\nu\right)}\,{}_2F_2\left(\begin{matrix}1+\frac{2c+p}{2|\beta|},\ 1+\frac{s+\xi}{\omega}\\ 2+\frac{2c+p}{2|\beta|},\ 1+\nu\end{matrix};Ax^{-2\beta}\right) \quad \text{(A.7)}$$

for $p < \frac{2|\beta|(s+\xi)}{\omega} - 2c$ (this restriction is due to the Gamma function $\Gamma\big((s+\xi)/\omega-(2c+p)/(2|\beta|)\big)$ in the first term since the Gamma function tends to infinity when its arguments are negative integers), and

$$\int_L^x y^{p+c-\frac{3}{2}-\beta} e^{\frac{A}{2}y^{-2\beta}} M_{\varkappa(s),\frac{\nu}{2}}\left(Ay^{-2\beta}\right)dy$$
$$= \frac{A^{\frac{1+\nu}{2}}x^{2c-2\beta+p}}{(2|\beta|+2c+p)}\,{}_2F_2\left(\begin{matrix}1+\frac{2c+p}{2|\beta|},\ 1+\frac{s+\xi}{\omega}\\ 2+\frac{2c+p}{2|\beta|},\ 1+\nu\end{matrix};Ax^{-2\beta}\right)$$
$$-\frac{A^{\frac{1+\nu}{2}}L^{2c-2\beta+p}}{(2|\beta|+2c+p)}\,{}_2F_2\left(\begin{matrix}1+\frac{2c+p}{2|\beta|},\ 1+\frac{s+\xi}{\omega}\\ 2+\frac{2c+p}{2|\beta|},\ 1+\nu\end{matrix};AL^{-2\beta}\right) \quad \text{(A.8)}$$

for $p > -2(c+|\beta|)$ if $L=0$ and for all real p with $L>0$.

Substituting these results into (A.6), along with $\frac{1+\nu}{2} - \varkappa(s) = 1 + \frac{s+\xi}{\omega}$, we obtain the explicit expression for the action of the resolvent operator on the power function x^p:

$$(\mathcal{R}_s x^p) = \frac{x^{\frac{1}{2}-c+\beta}e^{-\frac{A}{2}x^{-2\beta}}}{(\mu+b)\Gamma(1+\nu)}$$
$$\times\Bigg\{M_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\left[\frac{A^{\frac{1-2c-2p}{4|\beta|}-\frac{1}{2}}}{2|\beta|}\frac{\Gamma\left(1-\frac{1-p}{2|\beta|}\right)\Gamma\left(1+\frac{2c+p}{2|\beta|}\right)\Gamma\left(\frac{s+\xi}{\omega}-\frac{2c+p}{2|\beta|}\right)}{\Gamma\left(\frac{s+\xi}{\omega}+1-\nu\right)}\right.$$
$$-\frac{A^{\frac{1-\nu}{2}}x^{-2\beta-1+p}\Gamma(\nu)}{(2|\beta|-1+p)}\,{}_2F_2\left(\begin{matrix}1-\frac{1-p}{2|\beta|},\ 1+\frac{s+\xi}{\omega}-\nu\\ 2-\frac{1-p}{2|\beta|},\ 1-\nu\end{matrix};Ax^{-2\beta}\right)$$
$$\left.-\frac{A^{\frac{1+\nu}{2}}L^{2c-2\beta+p}\Gamma(-\nu)\Gamma\left(\frac{s+\xi}{\omega}+1\right)}{\Gamma\left(\frac{s+\xi}{\omega}+1-\nu\right)(2|\beta|+2c+p)}\,{}_2F_2\left(\begin{matrix}1+\frac{2c+p}{2|\beta|},\ 1+\frac{s+\xi}{\omega}\\ 2+\frac{2c+p}{2|\beta|},\ 1+\nu\end{matrix};AL^{-2\beta}\right)\right]$$
$$-\frac{M_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)}{W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)}W_{\varkappa(s),\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\times$$
$$\times\left[\frac{A^{\frac{1-2c-2p}{4|\beta|}-\frac{1}{2}}}{2|\beta|}\frac{\Gamma\left(1+\frac{p-1}{2|\beta|}\right)\Gamma\left(1+\frac{2c+p}{2|\beta|}\right)\Gamma\left(\frac{s+\xi}{\omega}-\frac{2c+p}{2|\beta|}\right)}{\Gamma\left(\frac{s+\xi}{\omega}+1-\nu\right)}\right.$$
$$-\frac{A^{\frac{1-\nu}{2}}L^{-2\beta-1+p}\Gamma(\nu)}{(2|\beta|-1+p)}\,{}_2F_2\left(\begin{matrix}1-\frac{1-p}{2|\beta|},\ 1+\frac{s+\xi}{\omega}-\nu\\ 2-\frac{1-p}{2|\beta|},\ 1-\nu\end{matrix};AL^{-2\beta}\right)$$

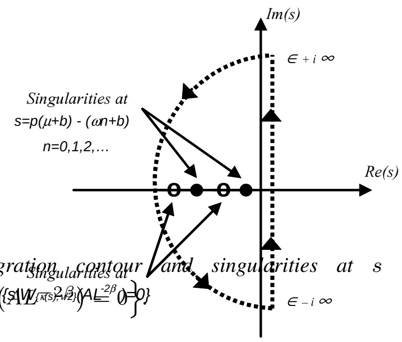

Fig. A.1 *Integration contour and singularities at* $s = p(\mu+b) - (b+\omega n)$ *and* $\left\{ s |\, W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right) = 0 \right\}$

$$-\frac{A^{\frac{1+\nu}{2}} L^{2c+p-2\beta}\,\Gamma(-\nu)\,\Gamma\left(\frac{s+\xi}{\omega}+1\right)}{(2|\beta|+2c+p)\,\Gamma\left(\frac{s+\xi}{\omega}+1-\nu\right)}\,{}_2F_2\left(\begin{matrix}1+\frac{2c+p}{2|\beta|},\ \frac{\omega+s+\xi}{\omega}\\ 2+\frac{2c+p}{2|\beta|},\ 1+\nu\end{matrix};AL^{-2\beta}\right)\Bigg]$$

$$-\frac{\Gamma(-\nu)\,\Gamma(1+\nu)\,M_{\varkappa(s),-\frac{\nu}{2}}\left(Ax^{-2\beta}\right)}{\Gamma(1-\nu)}\left[\frac{A^{\frac{1+\nu}{2}}x^{2c-2\beta+p}}{(2|\beta|+2c+p)}\,{}_2F_2\left(\begin{matrix}1+\frac{2c+p}{2|\beta|},\ \frac{\omega+s+\xi}{\omega}\\ 2+\frac{2c+p}{2|\beta|},\ 1+\nu\end{matrix};Ax^{-2\beta}\right)\right.$$

$$\left.-\frac{A^{\frac{1+\nu}{2}}L^{2c-2\beta+p}}{(2|\beta|+2c+p)}\,{}_2F_2\left(\begin{matrix}1+\frac{2c+p}{2|\beta|},\ \frac{\omega+s+\xi}{\omega}\\ 2+\frac{2c+p}{2|\beta|},\ 1+\nu\end{matrix};AL^{-2\beta}\right)\right]\Bigg\}. \qquad (A.9)$$

We investigate this expression as a function of the complex variable s. This function has three series of poles. To recover the expectation (A.4), we need to invert the Laplace transform by calculating the Bromwich contour integral on the right hand side of Eq. (A.4). We use the Cauchy Residue Theorem to reduce the Laplace inversion integral to the sum over all the residues at the poles of the integrand. Two series of poles lie along the negative part of the real axes and contribute non-zero residues to the calculation of the contour integral by the Cauchy Residue Theorem (see Figure A.1). The third series of poles also lie along the negative part of the real axes but have vanishing residues. The three series of poles and the corresponding residues are as follows.

(a) The Gamma function $\Gamma\left(\frac{s+\xi}{\omega}-\frac{2c+p}{2|\beta|}\right)$ has simple poles at $s = s_n = \omega\left(\frac{2c+p}{2|\beta|}-n\right)-\xi$, $n = 0,1,2,...$ The residues at these poles are calculated using the well-known result for the residues of the Gamma function at negative integers:

$$\mathrm{Res}_{z=-n}\Gamma(z) = (-1)^n/n!,\ \ n = 0,1,2,\ldots.$$

Calculating the residues at these poles produces the first sum in the expression (3.20).

(b) The second series of poles comes from the factor $1/W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)$. Recall that the Whittaker function $W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)$ has a series of zeros $\varkappa_n$, $n = 1,2,...$ described in Theorem 3.2 (see also Slater [24], p.105). These zeros lead to the poles of $1/W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)$ at $s = s_n$ such that $\varkappa(s_n) = \varkappa_n$, $n = 1,2,....$ Calculating the residues at these poles produces the second sum in the expression (3.20).

(c) Moreover, the Gamma function $\Gamma\left(\frac{s+\xi}{\omega}+1\right)$ has simple poles at $s = s_n = -\omega(n+1)-\xi$, $n = 0,1,2,....$ Calculating the residues at these poles, we find that each residue has the factor (in this case $\varkappa(s_n) = \nu/2+n+1/2$):

$$\left[\frac{M_{\frac{\nu}{2}+n+\frac{1}{2},\frac{\nu}{2}}\left(AL^{-2\beta}\right)}{W_{\frac{\nu}{2}+n+\frac{1}{2},\frac{\nu}{2}}\left(AL^{-2\beta}\right)}W_{\frac{\nu}{2}+n+\frac{1}{2},\frac{\nu}{2}}\left(Ax^{-2\beta}\right)-M_{\frac{\nu}{2}+n+\frac{1}{2},\frac{\nu}{2}}\left(Ax^{-2\beta}\right)\right].$$

However, when $k = \frac{\nu}{2} + n - \frac{1}{2}$, $n = 1, 2, ...$, the Whittaker functions $M_{k,\frac{\nu}{2}}(x)$ and $W_{k,\frac{\nu}{2}}(x)$ become linearly dependent and reduce to the generalized Laguerre polynomials (see Buchholz [8] p.214). Thus, these factors with the Whittaker functions vanish, and the contributions from these poles to the Laplace inversion vanish.

The results for the limiting case $L = 0$ (CDS) are obtained by observing that the ratio $M_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right)$ / $W_{\varkappa(s),\frac{\nu}{2}}\left(AL^{-2\beta}\right) \to 0$ in the limit $L \to 0$. □

B The Case with $\mu + b < 0$

The counterpart of the expression (3.20) for $\mu + b < 0$ is:

$$\mathbb{E}_x[e^{-\int_0^t h(X_u)du}\mathbf{1}_{\{T_L>t\}}(X_t)^p] = x^{\frac{1}{2}-c+\beta}e^{\frac{A}{2}x^{-2\beta}}\sum_{n=1}^{\infty}e^{-\lambda_n t}\,c_n\,N_n\,W_{\varkappa_n,\frac{\nu}{2}}\left(Ax^{-2\beta}\right), \tag{B.1}$$

where A, ξ, ν and ω are defined in (3.12), $\{\varkappa_n, n = 1, 2, \ldots\} = \left\{\varkappa \,\middle|\, W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right) = 0\right\}$, and N_n, λ_n, and c_n are defined as follows:

$$N_n = \sqrt{\frac{a^2|\beta|\Gamma(1/2+\nu/2-\varkappa_n)M_{\varkappa_n,\frac{\nu}{2}}\left(AL^{-2\beta}\right)}{\Gamma(1+\nu)\left[\frac{d}{d\varkappa}W_{\varkappa,\frac{\nu}{2}}\left(AL^{-2\beta}\right)\right]\Big|_{\varkappa=\varkappa_n}}},\ \lambda_n = -\omega\left(\frac{1-\nu}{2} - \varkappa_n\right) + \xi\,, \tag{B.2}$$

$$\begin{aligned} c_n = \frac{2N_n}{a^2} \times \Bigg[& \frac{A^{\frac{1-2p-2c}{4|\beta|}-\frac{1}{2}}\Gamma\left(\frac{2c+p}{2|\beta|}+1\right)\Gamma\left(\frac{p-1}{2|\beta|}+1\right)}{2|\beta|\Gamma\left(\frac{2c+2p-1}{4|\beta|}+\frac{3}{2}-\varkappa_n\right)} \\ & -\frac{A^{\frac{\nu+1}{2}}L^{2c-2\beta+p}}{2c+2|\beta|+p}\frac{\Gamma(-\nu)}{\Gamma\left(\frac{1-\nu}{2}-\varkappa_n\right)}\,{}_2F_2\left(\begin{matrix}\frac{2c+p}{2|\beta|}+1, & \frac{\nu+1}{2}+\varkappa_n \\ \nu+1, & \frac{2c+p}{2|\beta|}+2\end{matrix};-AL^{-2\beta}\right) \\ & -\frac{A^{\frac{1-\nu}{2}}L^{p-2\beta-1}}{p+2|\beta|-1}\frac{\Gamma(\nu)}{\Gamma\left(\frac{1+\nu}{2}-\varkappa_n\right)}\,{}_2F_2\left(\begin{matrix}\frac{p-1}{2|\beta|}+1, & \frac{1-\nu}{2}+\varkappa_n \\ 1-\nu, & \frac{p-1}{2|\beta|}+2\end{matrix};-AL^{-2\beta}\right)\Bigg]. \end{aligned} \tag{B.3}$$

The counterpart of the expression (3.21) for $\mu + b < 0$ is:

$$\begin{aligned} & \mathbb{E}_x[e^{-\int_0^t h(X_u)du}\mathbf{1}_{\{T_L>t\}}(X_t)^p] \\ & = A^{\frac{1-p}{2|\beta|}}x\,\Gamma\left(\frac{2c+p}{2|\beta|}+1\right) \times \sum_{n=1}^{\infty}e^{(\omega(1-\nu-n)-\xi)t}\frac{\left(\frac{1-p}{2|\beta|}\right)_{n-1}}{\Gamma(\nu+n)}L_{n-1}^{(\nu)}\left(Ax^{-2\beta}\right). \end{aligned} \tag{B.4}$$

The proofs are available upon request. The proofs for the case with $\mu + b < 0$ are actually easier than for the case with $\mu + b > 0$ presented in Appendix A since the function $f(x) = x^p$ is in the Hilbert space $L^2((L,\infty),\mathfrak{m})$ for $\mu + b < 0$ and, hence, one can write down the spectral expansion directly instead of inverting the Laplace transform (of course, inverting the Laplace transform by applying the Cauchy Residue Theorem leads to the same spectral expansion).

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